Five Year Integrated M.Sc. Examination, 2023-2024
Semester - VII
Code: PH-4-7-1

Time: Four Hours | Subject: Advanced Statistical Mechanics | Full Marks: 80

Questions are of values as indicated in the margin
Answer question number **one** and any **four** from the rest

1. Answer any **four** questions:

$4 \times 5 = 20$

(a) Calculate the partition function of two Fermions each of which can occupy any of the two energy levels of 0, ϵ and 2ϵ. Calculate the partition function if there are two Bosons instead of Fermions.

(b) Show that the Fermi energy E_F of electrons in a metal at $T = 0$ is given by ,

$$E_F = \frac{\hbar^2}{2m}\left(\frac{6\pi^2}{g_s}\frac{N}{V}\right)^{2/3} ,$$

where symbols have their usual meanings.

(c) Calculate the dispersion relation of the sound waves propagating through a monatomic chain.

(d) Draw the Fermi distribution function as $T = 0$ and $T > 0$. Physically explain the origin of linear temperature dependence of electronic specific heat ($C_V \sim T$) using the behaviour of Fermi distribution function at low T.

(e) Qualitatively explain the absence of magnetic ordered phase in 1D Ising chain at finite temperature by invoking the 'minimum free energy' argument.

(f) State about the conditions for the two systems to be in thermal, mechanical and chemical equilibrium.

2. (a) Calculate the density of states of photons and hence calculate the partition function of the photon gas.

(b) Using the partition function derive Planck's distrigasbution law and hence prove the Wien's displacement law i.e. $(\omega_{\max}/T) = constant$.

(c) Briefly explain the Cosmic Microwave Background Radiation. Without doing explicit calculation, show that the energy density of a photon gas varies as the fourth power of absolute temperature (T^4).

(2+3)+(3+2)+(2+3)=15

3. (a) Write grand canonical partition function for Bose gas and calculate the average occupation number $(n(\epsilon_p))$ of the p-th level with energy ϵ_p. Show that the total number of bosons in a system (N) can be expressed as $N = \sum_p n(\epsilon_p)$.

   (b) Hence show that the chemical potential of a Bose gas is either negative or zero. What is the maximum value of fugacity (z) for Bose gas?

   (c) Calculate the thermal de-Broglie wavelength for a system of N particles confined within a volume V at equilibrium temperature T. State the relation between inter-particle distance and thermal de-Broglie wavelength for the applicability of quantum statistical mechanics to describe this system of particles.

(3+2)+5+(3+2)=15

4. (a) Calculate the average number of particles (N) of a Bose gas at low temperature limit. Show that below the critical temperature T_c, the number of particle occupies the ground state is given by

$$\frac{n_0}{N} = 1 - \left(\frac{T}{T_c}\right)^{3/2} .$$

   Using $g_n(1) = \zeta(n)$, obtain an expression for T_c. Here all notations have their usual meaning. Bose function $g_n(z)$ is defined as

$$g_n(z) = \frac{1}{\Gamma(n)} \int_0^\infty dx \frac{x^{n-1}}{z^{-1}e^x - 1} .$$

   (b) By using diagrammatic method, calculate the energies and the degeneracies of ground state, first excited states and second excited states of 2D Ising chain on square lattice at low temperature.

(3+4)+(2+3+3)=15

5. (a) Briefly state the postulates of Landau theory of second order phase transition.

   (b) Using the postulates show that the Landau free energy functional for ferromagnetic system can be expressed as

$$\mathcal{L} = at\,\phi^2 + \frac{1}{2}b\,\phi^4\,,$$

   where a and b are two temperature independent parameters, T_c is the Curie temperature, ϕ is the order parameter and $t = (T - T_c)/T_c$. Calculate the equilibrium values of ϕ for both $t < 0$ and $t > 0$.

   (c) Draw the Landau free energy functional for different temperature regimes and show that it can represent continuous phase transition at $T = T_c$.

   (d) How does Landau free energy get modified in presence of external field?

2+(2+3)+(3+2)+3=15

6. (a) Write the 1D Ising model Hamiltonian and explain all the terms.

   (b) Construct the transfer matrix of 1D Ising model **in presence of an external magnetic field** (h) and calculate its eigenvalues?

   (c) Express the partition function of 1D Ising model as a function of eigenvalues of the aforementioned transfer matrix. Show that the magnetization is zero in absence of external magnetic field.

3+(3+3)+(3+3)=15